What motor, screw and gearing should I choose?

Document version 1.2

Changes from version 1.1:

- Corrected an error in the equation for power. Thanks to Jay C for pointing this out!
- Added a note about the Wikipedia page on the Pound on page 4.

Changes from version 1.0:

- Corrected several spelling errors (thanks to Graham for pointing these out).
- Emphasised constant power/constant torque for clarity in the “Back to the motors” section (again thanks to Graham for the idea).
- Explained the difference between nominal and full speed of a servo motor (in the “Back to the motors” section).
- A bit more on different screw leads and stepper motors in the “Selecting the right screw with regards to speed” section.

The short answer

For a smaller wood and plastic cutting machine (and maybe – maybe! – some light aluminum), several people on CNCzone seems to have had success with 100-200 oz-in (0.7-1.4 Nm) stepper motors direct driving a 5 turns/inch (5 mm lead) ACME (trapezoidal) screw.

The long answer starts here!

I'd say this is best done in a repetitive process (step 1 & 2 described in detail below):

1) Decide on cutting speed (feed rate), cutting force, and jog speed. The first time, just choose something that will make you happy.
2) Calculate what motor, gearing and screw is needed to get above values.
3) Scout out manufacturer's homepages, Ebay and any other source you can think of, to see if motors etc. with the specs calculated in step 2 are available within your budget.
4) Adjust your expectations and/or budget and start all over again.

The first time you do the above, it will take time. Each new time, it will be much quicker and you will soon be an expert at calculating speeds, forces and torques!

Before we start

Before we start we need to define a few very basic units. These are units we all know well, but it might be a good idea to have a clear definition of them anyway. All quantities that will be used in this text will be defined with both SI and imperial units, including conversion factors. However, most if not all of the *calculations* that follows will be done with SI units – they almost never require any conversion factors and thus shows the physical principles behind the calculation in a nice way.

IMPORTANT: All designations shown are for the SI units (for example, 's' is the designation for distance in meters, not inches); this means most equations using the SI designations will NOT work for imperial units! You'll have to convert your quantities to SI units before using the equations!

Now these first two are easy:

Distance, **s**

SI unit: **1 m** (meter)
Imperial unit: **1 in** (inch) = 0.0254 m

Sometimes I will also use the unit 1 mm (millimeter) which is 1/1000 m or about 1/25 in.

(To go from imperial to SI: multiply with the factor like this: 2 in = 2*0.0254 = 0.0508 m. To go from SI to imperial: divide with the factor like this: 0.5 m = 0.5/0.0254 ≈ 19.69 in.)

Time, t

SI unit: **1 s** (second)
Other units: **1 min** (minute) = 60 s

1) Feed rates and forces

We need three more units to get through this. You should read through these definitions and make sure you understand them; for example, what is the difference between mass and weight, and how are they related?

If you get through and understand these and the next section's three unit definitions, you will have a good understanding of the mechanics of a CNC machine.

Velocity (speed), v

SI unit: **1 m/s** (meter per second)
Imperial unit: **1 IPM** (inch per minute) ≈ 0.0004233 m/s

Let's say you happily pick 0.05 m/s (≈ 118 IPM) as max jog speed, and 0.01 m/s (≈ 24 IPM) max cutting speed.

Mass, m

SI unit: **1 kg** (kilogram)
Imperial unit: **1 lb (pound)** ≈ 0.4536 kg or **1 oz** (ounce) ≈ 0.02835 kg

> The more mass an object has, the more it resists acceleration/deceleration. Mass interacts with gravity to give weight (observe that mass and weight are not the same thing– you might be weightless in space, but you still have the same mass – see Force below).

Force, F

SI unit: **1 N** (newton)
Imperial unit: **1 lbf** (pound-force) ≈ 4.448 N or **1 ozf** (ounce-force) ≈ 0.2780 N

> A force is needed to accelerate or decelerate a mass. The *weight* of an object is the force developed by the gravity between the masses of the earth and the object. (Actually there's a force of gravity between all objects that have mass, but it's so tiny we don't notice.)

You might think 200 N (≈ 720 ozf) would be an adequate cutting and accelerating force.

Note: I would recommend a reading of the Wikipedia page on the Pound (http://en.wikipedia.org/wiki/Pound) for an explanation of why one unit, the pound, is used to describe the two very different quantities mass and force.

2) Calculating motor, gearing and screw values

There's an important difference between servo motors and steppers, besides the electronics required to run them. The mechanical design requirements are different for the two types of motors, so we have to make this difference clear. To be able to do so, more physics is needed! Again, spend some time with the definitions until you feel you have fully understood them. Hang on, these are the last three!

Rotational speed

Unit: **1 r/s** (revolution per second). Let's designate this **n**.
1 RPM (revolution per minute) = 1/60 r/s

We all know what RPM is. Revolutions per second should be clear too. (For the sake of completeness, I will mention that there's also an SI unit for measuring rotational speed. It's called "angular velocity", has designation ω (omega), and unit 1 radian/s. If you know trigonometry, then perhaps this unit makes sense. 1 r/s = 1/(2*π) rad/s.)

Torque or "moment of force", **M**

SI unit: **1 Nm** (newton meter)
Imperial unit: **1 oz*in** (oz-in) ≈ 0.007062 Nm

Torque forces something to rotate, and is "force acting from a distance". Torque is the *product* of force and distance (so writing oz/in, ounce per inch, is *wrong*). Think about opening a door: you force it to rotate on its hinges by pushing (applying a force) on the handle. No torque is produced if you were to push directly on the hinges, since the distance from the point of rotation is zero.

$$M = F \cdot s$$

Motor strength is measured in units of torque. (Some manufacturers measure their motors' strength in "kg-cm" or similar. This is a nonsense unit, that shows only that the manufacturer has not fully understood the difference between force and mass. What is meant is the force of gravity (weight) of the mass 1 kg here on earth, times 1 cm. The force of gravity on 1 kg at sea level is about 9.81 N, and one cm is 1/100 m, so 1 "kg-cm" can be "converted" to 0.0981 Nm.)

Power, **P**

SI unit: **1 W** (watt)
Imperial unit: **1 HP** (imperial horse power) ≈ 745.7 W

Power is (in a CNC context) force times speed, torque times rotational speed, or voltage times current. That is, power measures how fast something is done, times how difficult it is to do it.

$$P = F \cdot v = M \cdot n \cdot 2\pi = M \cdot \omega = U \cdot I$$

(U*I refers to electrical power; voltage times current.)

Motor power measures either how fast a motor can rotate while producing a certain amount of torque ("output power" or "mechanical power"), or how much "electricity" the motor draws ("input power"). Output power is always less than input power (usually 80-95 percent for an electrical motor).

Back to the motors.

Approximately, a stepper motor have a **constant power** output, which means that when speed increases, torque drops (P = 2*pi*M*n means that if speed increases, torque must drop to keep power constant). So no matter what speed you run it at, you get about the same power output from the stepper (not true at the extremes: when the motor is stalled – n or ω is zero – then power output have to be zero too; look at the definition of power above).

A servo motor, on the other hand, has an (almost) **constant torque** up to nominal speed. This means that maximum output power for a servo motor is developed at the motor's nominal to full speed. (The nominal speed is the top speed at which you can still use the motor's full torque, full speed is the maximum speed the motor can take. For one of my servo's nominal speed is 3000 RPM, full speed is 4500 RPM. Power output is more or less constant from nominal to full speed.)

Since lots of power means lots of either torque or speed, or both, we want to maximize power. We do this by selecting the best screw (and gearing).

Selecting the right screw with regards to speed

You chose 0.05 m/s jog speed in step 1. Let's first look at what bearing this has on the screw (pun intended).

A common metric screw is one with a *lead* of 0.005 m (= 5 mm). This means the screw moves the nut 0.005 m each turn. A common imperial screw is one with a *pitch* of 5 turns/in. This means the screw requires 5 turns to move the nut one inch. To convert imperial pitch to metric lead (for use in the equations that follow), divide 0.0254 with the pitch number. Example with the imperial screw above: 0.0254/5 ≈ 0.005, which shows that this screw has almost the same lead (or pitch) as the metric one above.

Now if your machine shall have a top (jog) speed of 0.05 m/s and the nut moves 0.005 m/turn, then the screw will need to rotate 0.05/0.005 = 10 r/s to get this speed. So the equation for n (screw rotational speed in r/s) is:

$n = \frac{v}{s}$, where s is the lead of the screw and the other designations are as defined above.

If you are using a servo motor it will probably have a higher nominal speed than the above (10 r/s = 600 RPM). Remembering the characteristics of servo motors, if your servo motor's nominal speed is 3000 RPM, then we will only be able to use 600/3000 = 20% of the motor's speed, and thus only 20% of its power capacity! This is not good. We can fix this in two ways: by gearing down and/or by using a screw with shorter lead. Both have the effect of giving more force and less speed at the nut, for the same motor speed.

So if we gear down 5:1 then at 3000 RPM motor speed (and thus 100% motor power), the screw will still rotate at 600 RPM (3000/5), but now with 5 times as much torque! (Remember, power is torque times rotational speed, so if power increases 5 times and speed stays the same, then torque must have increased 5 times.) So it is quite important to use correct gearing and screw lead with a servo motor!

If you use a stepper, this is easier. Power is constant, so gearing up or down will not change much. I guess it's because of this most stepper driven machines simply skips gearing and direct drive the screws.

However, with a high lead, you lose resolution since the nut will move a greater distance for each step of the stepper. Also, if the motor is at standstill and you command a step, a higher lead will require a higher acceleration of the nut and attached table/gantry (it will move a longer distance during the time of the step than with a lower lead). This might lead to lost steps – see Acceleration below.

One thing to notice from the above is that gearing and screw lead is closely related; in fact, halving the screw lead (thus halving linear speed) have exactly the same effect as gearing down 2:1 (which halves the screw's rotational speed and thus linear speed too).

Selecting the right screw with regards to force

We saw above how screw lead influences speed. But screw lead also affect how much torque is needed to get a certain force at the nut. Here is the equation:

$M = F \cdot \frac{s}{2\pi} \cdot \frac{1}{\eta}$. Again, s is screw lead. η is the *efficiency* of the screw, a very important property of screws, as we will soon be aware.

The efficiency of the screw determines how much of the input torque is converted to output force. Ball screws excel here, with $\eta = 0.9$ or there about. A good quality ACME/trapezoidal screw has $\eta = 0.4$ or so. All thread (or metric M thread) probably has η between 0.1 and 0.2. So a ball screw might give up to 9 times as much force than all thread with exactly the same motor! An ACME screw might be a good compromise (probably has the best performance/price ratio).

If you look at the equation, you can see that torque (M) is proportional to force (F) times screw lead (s). So for any given output force, if you halve the lead you will also halve the required input torque for that output force. Again we see that gearing and lead is closely related; you can also halve the required motor torque by gearing down 2:1.

To repeat: halve the lead and you get twice the force and half the speed from the screw, using the same motor (notice that power is constant: 2 times ½ = 1). Gear down 2:1 and exactly the same thing happens.

This also means that the efficiency of the screw is important for speed. If you exchange all thread for a ball screw with the same lead, you suddenly get a lot of force that you might not need. It is probably a lot better to select a ball screw with higher lead which would give you more speed while keeping the force of the all thread. The choice is yours: if the efficiency is 8 times higher, you can get 8 times the speed, 8 times the force, or a combination (i.e. 2 times the force and 4 times the speed) by selecting the right screw lead and gear ratio.

Back to our example machine

Servo motor: with jog speed 0.05 m/s and nominal motor speed 3000 RPM we might want to select an ACME screw with s = 0.004 m. Screw rotational speed n = v/s = 0.05/0.004 = 12.5 r/s = 750 RPM. So we need to gear down 4:1 (3000/750) to fully utilize the motor. We wanted a cutting force of 200 N, so torque at screw = 200*0.004/(2*pi)*1/0.4 ≈ 0.318 Nm, or after gearing down four times 0.080 Nm.

This is a very weak servo motor indeed (power output = 0.080*3000/60*2*pi ≈ 25 W). We might want to select a 100 W, 3000 RPM servo motor instead. Four times the motor power would let us reduce the gearing to 2:1, thus doubling the jog speed to 0.1 m/s (some 236 IPM), while also doubling our force to 400 N – 4 times the motor power at the same speed means four times the torque, so after reducing the gearing with a factor 2 we have 4/2 = 2 times the torque left.

Because servos are constant torque, this force (the cutting force) is available all the way *up to nominal speed.*

Stepper motor: with a cutting speed of 0.01 m/s and with the same AMCE screw as above (lead of 0.004 m), the screw speed n is 0.01/0.004 = 2.5 r/s = 150 RPM. We saw above that 0.318 Nm was required to get the 200 N of force at the nut. You will have to look at the speed-torque curve in the stepper's data sheet to see that the stepper is capable of this torque at 150 RPM. (I just checked a random data sheet and it looks like a stepper with 0.4 or 0.5 Nm holding torque (60 oz-in) should be able to handle this.)

Jog speed is more difficult; the force needed to accelerate your machine (gantry, table, Y carriage etc) – and to overcome friction to keep it moving – is dependent on the mass you want to move and the friction in your linear bearings, among other things. If you can estimate this force, you can again look at the data sheet to see that the stepper can handle this at the jogging speed.

Acceleration

Finally, we have to look at acceleration. (This part is especially important for steppers.)

The force developed at the screw's nut is not only used for cutting, but also to overcome friction and *accelerate and brake* your machine. This is often overlooked. I said before that there were no more unit definitions, but that was a lie.

Acceleration, a

SI unit: **1 m/s²** (meter per second squared).

The unit of speed is 1 m/s, and acceleration measures how fast this change per second. How many m/s per second becomes m/s/s or m/s^2.

$$a = \frac{F}{m}$$

We see that acceleration is dependent on both force and mass. The more mass, the slower the acceleration, and the more force, the faster the acceleration.

I will not elaborate on this other than to say that if you command a fast acceleration, make sure your stepper has enough torque to produce the required force (in combination with the screw and – if applicable – gearing). Otherwise it will lose steps. The same thing happens if you run your stepper so fast that its torque and therefore the force at the nut drops low enough not to overcome the friction of your bearings (or other opposing forces).

A servo motor will fall behind some if you command too fast an acceleration, but servos have spare torque (peak torque as opposed to nominal or continuous torque) and will catch up (unless it falls too far behind and faults out).

There is also something called rotational inertia which is the rotational equivalence of mass from an acceleration point of view. It takes torque to make a screw start to spin (just as it takes force to make a mass start to move), and it can take a lot of torque if the screw is thick because it then has lots of rotational inertia. So if you want to calculate acceleration, you will need to learn about this first.

Final notes

All calculations here assume 100% efficiency everywhere except at the screw, and disregards all forms of friction. So you will need some additional power from your motors to get the expected result. Depending on the quality of your guides, bearings and how well adjusted they are, this will vary, but a rough estimation might be that 50% extra torque is "safe".

Last but very important: for all vertical axes you have to add the force of gravity (weight) of what you are moving to the force required at the nut (unless you use some kind of counterweight). This is calculated as $F_g = 9.81 \cdot m$. So a 10 kg Z carriage (incl. router or other tool) would need 98.1 N of force at the nut, just to keep still. As you can see this is considerable. You have to add this to the force needed for accelerating the Z carriage upwards. But you can subtract is from the force needed to push the tool into the work in the Z direction.

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